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A Class of Continuous Implicit Seventh-eight method for solving y' = f(x, y) using power series

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Abstract— *In this article, we develop a continuous implicit seventh-eight method using interpolation and collocation of the approximate solution for the solution of y' = f(x, y) with a constant step-size. The method uses power series as the approximate solution in the derivation of the method. The independent solution was then derived by adopting block integrator. The properties of the method was investigated and found to be zero stable, consistent and convergent. The integrator was tested on numerical examples ranging from linear problem, Prothero-Robinson Oscillatory, Growth Model and Sir Model. The results show that the computed solution is closer to the exact solution and also, the absolutes errors perform better than the existing methods.*

Keywords*— *Seventh-eight method, Continuous Implicit method, Power Series, y' = f(x, y) P-stable, Growth Model, SIR model, Prothero-Robinson Oscillatory, Convergent.

Mathematical Subject Classification: 65L05, 65L06, 65L08

I. INTRODUCTION

Numerical analysis predates the invention of modern computers by many centuries. Linear interpolation was already in use more than 2000 years ago. Many great mathematicians of the past were preoccupied by numerical analysis, and this is obvious from the names of important algorithms like Newton's method, Lagrange interpolation polynomial, Gaussian elimination and Euler's method. Despite the long and rich history of numerical analysis, "modern" numerical analysis is characterized by the synergy of the programmable electronic computer, mathematical analysis, and the opportunity and need to solve large and complex problems in applications. The need to advance in applications such as ballistics prediction, neutron transport, and non-steady multidimensional fluid dynamics motivated the development of the computer which depended strongly on advances in numerical analysis and mathematical modelling.

1.1 Aim and Objectives

The aim of this research work is to develop continuous method via fractional-step methods by using power series polynomial as basic functions for solving IVPs in ODEs. The objectives are to.

i. develop hybrid block methods through continuous implicit seventh-eight method
ii. analyzing the basic properties of the method; zero stability, order, convergence and region of absolute stability
iii. implementing a self-staring implicit methods without the rigor of developing predictors
iv. developing computer codes to implement the method derived using MATLAB
v. examine the exact solution and the computed solution, if they are in agreement
vi. compute the time taken for computation in seconds

1.2 Significance of the Study

The study is significant for the following reasons

1. This work centered on the numerical solution of first order ordinary differential equations of initial

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value problem. Hence, the differential equations whose order is more than one are not considered.

2. The proposed method makes use of power series polynomial, other polynomials are not considered in this work.
3. Throughput this research work, linear fractional-step method is considered, Non hybrid points are not considered.
4. Most of the modeled problems in ordinary differential equations do not have analytical or theoretical solution, hence the use of numerical solution is very important

II. LITERATURE REVIEW

This work is based on numerical solution of first order initial value problems of ordinary differential equation, which is of the form

$$y' = f(x, y),\ y(a) = \eta,\ a \le x \le b \tag{1}$$

where f is continuous with the interval of integration. It is assumed that f satisfies Litchitz condition which guarantees the existence and uniqueness of solution (1).

Countless real life problems in sciences, engineering, biology and social sciences are model of first order ordinary differential equations. For instance, Environmental Models such as SIR Model, Growth model, Mixture model just to mention few. Olver (2008) and Kandasamy *et al.* (2005), reported that equation (1) is used in simulating the growth of populations, trajectory of a particle, simple harmonic motion, deflection of a beam etc. With the advent of the modern high speed electronic digital computers, the numerical integrators have been successfully applied to study problems in mathematics, engineering, computer science and physical sciences such as physics, biophysics, atmospheric sciences and geosciences, Jain et al. (2007)

Many numerical integration schemes to generate the numerical solution to problems of the form (1) have been proposed by several authors. The one-step methods include those developed by Fatunla (1994), Aashikpelokhai (1991), Ayinde and Ibijola (2015), and Kama and Ibijola (2000), Ibijola (1998), Ogunrinde (2010) etc . These methods were constructed by representing the theoretical solution $y(x)$ to (1) in the interval, $[x_n, x_{n+1}]\ n \ge 0$ by linear and non-linear polynomial interpolating functions.

Furthermore, authors like Butcher (2003), Sunday et al. (2015), Lukman and Olaoluwa, Areo *et al.* (2011), Ibijola, Skwame and Kumleng (2011), Olangegan et al, (2015), Areo and Omojola (2017), Adeniran and Longe (2019), Adeniyi et al. (2006), Onumanyi et al, (1994), Sunday and Odekunle (2012), Badmus (2014), Jator (2014), Kayode and Adeyeye (2013) and Ibijola et al. (201) have all proposed linear multistep methods (LMMs) to generate numerical solution to (1). These authors proposed methods in which the approximate solution ranges from power series, Chebychev's, Lagrange's, Lucas polynomial, Legendre Polynomial and Laguerre's polynomials. In spite of the improvement made by diverse researchers in developing schemes that solve (1), the order of the schemes are low resulting in lower accuracy when used to solve problems which is a setback. Hence, the setbacks by previous works motivated a need to develop a seventh-eight method for solving first order ordinary differential equations unlike other methods which is one-step or multi-step. The method will be analysed and test on some numerical examples to test the accuracy and efficiency of the new method in order to ascertain how reliable it is.

III. MATERIALS AND METHODS

The power series of the form (2) is considered

$$y(x) = \sum_{n=0}^{\delta+\sigma-1} a_n p_n(x), \tag{2}$$

The first derivative of (2) is given by (3)

$$y'(x) = \sum_{n=1}^{\delta+\sigma-1} a_n p'_n(x), \tag{3}$$

as an approximate solution to equation (1) where $p_n(x) = x^n$,. δ and σ are the number of distinct collocation and interpolation points respectively. Substituting the first derivative of (3) into (1) gives

$$f(x, y(x),) = \sum_{n=1}^{c+i-1} a_n p'_n(x), \tag{4}$$

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Collocating (4) at $x=0\left(\frac{1}{8}\right)\frac{7}{8}$ and interpolating (3) at $x=\frac{3}{4}$ leads to a system of nine equations which is solved with the aid of Maple Mathematical Software version 18.0 to obtain $a_n,\ n=0,1,\cdots,8$. The a_n's obtained are then substituted into (3) to obtain the continuous form of the method

$$y(x)=\alpha_{\frac{3}{4}}(t)y_{n+\frac{3}{4}}+h\begin{bmatrix}\beta_0(t)f_n+\beta_{\frac{1}{8}}(t)f_{n+\frac{1}{8}}+\beta_{\frac{1}{4}}(t)f_{n+\frac{1}{4}}+\beta_{\frac{3}{8}}(t)f_{n+\frac{3}{8}}+\beta_{\frac{1}{2}}(t)f_{n+\frac{1}{2}}+\beta_{\frac{5}{8}}(t)f_{n+\frac{5}{8}}\\+\beta_{\frac{3}{4}}(t)f_{n+\frac{3}{4}}+\beta_{\frac{7}{8}}(t)f_{n+\frac{7}{8}}\end{bmatrix}\tag{5}$$

where α_n and β_n are continuous coefficients.

$$\alpha_{\frac{3}{4}}(t)=1$$

$$\beta_0(t)=-\frac{1}{30240}\left(6144h^4t^4-12288h^3t^3+8192h^2t^2-2048ht+123\right)(4ht-1)^2(4ht-3)^2$$

$$\beta_{\frac{1}{8}}(t)=\frac{1}{3780}\left(86016h^6t^6-202752h^5t^5+176128h^4t^4-69312h^3t^3+11328h^2t^2-216ht-81\right)(4ht-3)^2$$

$$\beta_{\frac{1}{4}}(t)=-\frac{1}{3360}\left(229376h^6t^6-507904h^5t^5+399360h^4t^4-133120h^3t^3+15728h^2t^2+24ht+9\right)(4ht-3)^2$$

$$\beta_{\frac{3}{8}}(t)=\frac{1}{1890}\left(215040h^6t^6-445440h^5t^5+317440h^4t^4-92448h^3t^3+9528h^2t^2-136ht-51\right)(4ht-3)^2$$

$$\beta_{\frac{1}{2}}(t)=-\frac{1}{30240}\left(3440640h^6t^6-6635520h^5t^5+4311040h^4t^4-1155072h^3t^3+118032h^2t^2+216ht+81\right)(4ht-3)^2$$

$$\beta_{\frac{5}{8}}(t)=\frac{1}{420}\left(28672h^6t^6-51200h^5t^5+30720h^4t^4-8000h^3t^3+736h^2t^2-24ht-9\right)(4ht-3)^2$$

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$$\beta_{\frac{3}{4}}(t) = -\frac{1}{30240}(4ht-3)\left(2752512h^7t^7 - 6586368h^6t^6 + 5955584h^5t^5 - 2586624h^4t^4 + 545088h^3t^3 - 47696h^2t^2 - 492ht - 369\right)$$

$$\beta_{\frac{7}{8}}(t) = \frac{4}{945}t^2h^2\left(768h^4t^4 - 1152h^3t^3 + 640h^2t^2 - 156ht + 15\right)(4ht-3)^2$$

The continuous method (5) is used to generate the main method. That is, we evaluate at $x = x_{n+\frac{7}{8}}$

$$y_{n+\frac{7}{8}} = y_{n+\frac{3}{4}} + \frac{1}{967680}h\left(1375f_n + 123133f_{n+\frac{1}{2}} + 41499f_{n+\frac{1}{4}} - 11351f_{n+\frac{1}{8}} + 139849f_{n+\frac{3}{4}} - 88547f_{n+\frac{3}{8}} - 121797f_{n+\frac{5}{8}} + 36799f_{n+\frac{7}{8}}\right) \tag{6}$$

Continuous method (5) is also used to generate the additional method at the non-interpolation points. That is, we evaluate at $x = x_{n+\frac{1}{8}}, x_{n+\frac{1}{4}}, x_{n+\frac{3}{8}}, x_{n+\frac{1}{2}}$ and $x = x_{n+\frac{5}{8}}$

$$y_n = y_{n+\frac{3}{4}} - \frac{1}{1120}h\left(41f_n + 27f_{n+\frac{1}{2}} + 27f_{n+\frac{1}{4}} + 216f_{n+\frac{1}{8}} + 41f_{n+\frac{3}{4}} + 272f_{n+\frac{3}{8}} + 216f_{n+\frac{5}{8}}\right) \tag{7}$$

$$y_{n+\frac{1}{8}} = y_{n+\frac{3}{4}} + \frac{5}{193536}h\left(55f_n - 4475f_{n+\frac{1}{2}} - 5805f_{n+\frac{1}{4}} - 1871f_{n+\frac{1}{8}} - 1871f_{n+\frac{3}{4}} - 4475f_{n+\frac{3}{8}} - 5805f_{n+\frac{5}{8}} + 55f_{n+\frac{7}{8}}\right) \tag{8}$$

$$y_{n+\frac{1}{4}} = y_{n+\frac{3}{4}} - \frac{1}{3780}h\left(332f_{n+\frac{1}{2}} + 171f_{n+\frac{1}{4}} - 4f_{n+\frac{1}{8}} + 171f_{n+\frac{3}{4}} + 612f_{n+\frac{3}{8}} + 612f_{n+\frac{5}{8}} - 4f_{n+\frac{7}{8}}\right) \tag{9}$$

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$$y_{n+\frac{3}{8}} = y_{n+\frac{3}{4}} + \frac{1}{35840} h \left(13 f_n - 3897 f_{n+\frac{1}{2}} + 513 f_{n+\frac{1}{4}} - 117 f_{n+\frac{1}{8}} - 1685 f_{n+\frac{3}{4}} - 2777 f_{n+\frac{3}{8}} - 5535 f_{n+\frac{5}{8}} + 45 f_{n+\frac{7}{8}} \right) \quad (10)$$

$$y_{n+\frac{1}{2}} = y_{n+\frac{3}{4}} + \frac{1}{30240} h \left(5 f_n - 1153 f_{n+\frac{1}{2}} + 135 f_{n+\frac{1}{4}} - 40 f_{n+\frac{1}{8}} - 1363 f_{n+\frac{3}{4}} - 208 f_{n+\frac{3}{8}} - 4968 f_{n+\frac{5}{8}} + 32 f_{n+\frac{7}{8}} \right) \quad (11)$$

$$y_{n+\frac{5}{8}} = y_{n+\frac{3}{4}} + \frac{1}{967680} h \left(351 f_n + 44797 f_{n+\frac{1}{2}} + 11547 f_{n+\frac{1}{4}} - 2999 f_{n+\frac{1}{8}} - 47799 f_{n+\frac{3}{4}} - 26883 f_{n+\frac{3}{8}} - 101349 f_{n+\frac{5}{8}} + 1375 f_{n+\frac{7}{8}} \right) \quad (12)$$

The methods derived in equation (6) to (12) are combined and implemented as a block in solving numerical problems of (1).

IV. ANALYSIS OF THE METHOD

4.1 Order and Error Constant of the Block Method

We adopt the method proposed by Fatunla (1994), Lambert (1973) and Henrici (1962) to obtain the order of the method obtained from (6-12) as $(8,8,8,8,8,8,8)^T$ and Error constants as

$$\left[-6.97e^{-11}, -5.47e^{-11}, -6.14e^{-11}, -5.62e^{-11}, -6.29e^{-11}, -4.79e^{-11}, -1.18e^{-11} \right]^T \quad (13)$$

4.2 Zero Stability of the Block Method

A block method is said to be zero stable, if the roots

$$\det\left[\lambda A^{(0)} - A^{(i)} \right] = 0 \quad (14)$$

of the first characteristic polynomial satisfy $|\lambda| \le 1$ and for the roots with $|\lambda| \le 1$, the multiplicity must not exceed the order of the differential equation.

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$$A=\left[z\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{pmatrix}-\begin{pmatrix}0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\end{pmatrix}\right]=0 \tag{15}$$

$$A=z^{6}(z-1)=0, z=0,0,0,0,0,0,0$$

Hence the block is zero stable.

4.3 Consistency of the Method

Our new developed block method is consistent since the order of each of the method is greater than 1.

4.4 Convergence

According to Lambert, 1973, Brown 1977 and Chollom et al. (2007) the necessary and sufficient condition for a linear multistep method to be convergent is for it to be consistent and zero stable.

Hence Convergence = Consistency + Zero-stability.

Hence the new block method is convergent

4.5 Region of Absolute Stability of the Proposed Method

According to Ibijola et al. (2011), Areo and Omojola (2017), the stability matrix is expressed as

$$M(z)=V+zB(M-zA)^{-1}U \tag{16}$$

together with the stability function

$$p(\eta,z)=\det(\eta I-M(z)) \tag{17}$$

We express the block method obtained from (6-12) in the form

$$\begin{bmatrix} Y \\ \hline Y_{i+1}\end{bmatrix}=\begin{bmatrix} A & U \\ \hline B & V\end{bmatrix}\begin{bmatrix} hf(y) \\ \hline Y_{i-1}\end{bmatrix} \tag{18}$$

where

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$$U := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} : \quad V := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$$A := \begin{bmatrix} \frac{139849}{967680} & -\frac{4511}{35840} & \frac{123133}{967680} & -\frac{88547}{967680} & \frac{1537}{35840} & -\frac{11351}{967680} & \frac{275}{193536} \\ \frac{733}{3780} & -\frac{71}{3360} & \frac{17}{210} & -\frac{1927}{30240} & \frac{13}{420} & -\frac{29}{3360} & \frac{1}{945} \\ \frac{1359}{7168} & \frac{1377}{35840} & \frac{5927}{35840} & -\frac{3033}{35840} & \frac{1377}{35840} & -\frac{373}{35840} & \frac{9}{7168} \\ \frac{181}{945} & \frac{1}{35} & \frac{223}{945} & -\frac{53}{3780} & \frac{1}{35} & -\frac{8}{945} & \frac{1}{945} \\ \frac{36725}{193536} & \frac{775}{21504} & \frac{4625}{21504} & \frac{13625}{193536} & \frac{1895}{21504} & -\frac{275}{21504} & \frac{275}{193536} \\ \frac{27}{140} & \frac{27}{1120} & \frac{17}{70} & \frac{27}{1120} & \frac{27}{140} & \frac{41}{1120} & 0 \\ \frac{25039}{138240} & \frac{343}{5120} & \frac{20923}{138240} & \frac{20923}{138240} & \frac{343}{5120} & \frac{25039}{138240} & \frac{5257}{138240} \end{bmatrix}$$

$$B := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{5257}{138240} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{41}{1120} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{265}{7168} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{139}{3780} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{265}{7168} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{41}{1120} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5257}{138240} \end{bmatrix} : \quad Y = \begin{bmatrix} y_n \\ y_{n+\frac{1}{8}} \\ y_{n+\frac{1}{4}} \\ y_{n+\frac{3}{8}} \\ y_{n+\frac{1}{2}} \\ y_{n+\frac{5}{8}} \\ y_{n+\frac{7}{8}} \end{bmatrix}, \quad f(y) = \begin{bmatrix} f_n \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{3}{8}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{5}{8}} \\ f_{n+\frac{7}{8}} \end{bmatrix}, Y_{i-1} = \begin{bmatrix} y_{n+\frac{1}{8}} \\ y_n \end{bmatrix}, Y_{i+1} = \begin{bmatrix} y_{n+\frac{1}{8}} \\ y_{n+\frac{7}{8}} \end{bmatrix} \quad (19)$$

The elements of the matrices A, B, U and V are substituted and computing the stability function with Maple software yield the stability polynomial of the method which is then plotted in MATLAB environment with the Newton-Raphson Method where

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$N = 400$ and tol= 10^{-12} to produce the required absolute stability region of the method as shown in the figure 1 below. The graph is shows the region in which the method is absolutely stable. We conclude that the method is P-stable since the interval of periodicity lies between (0, 2.0).

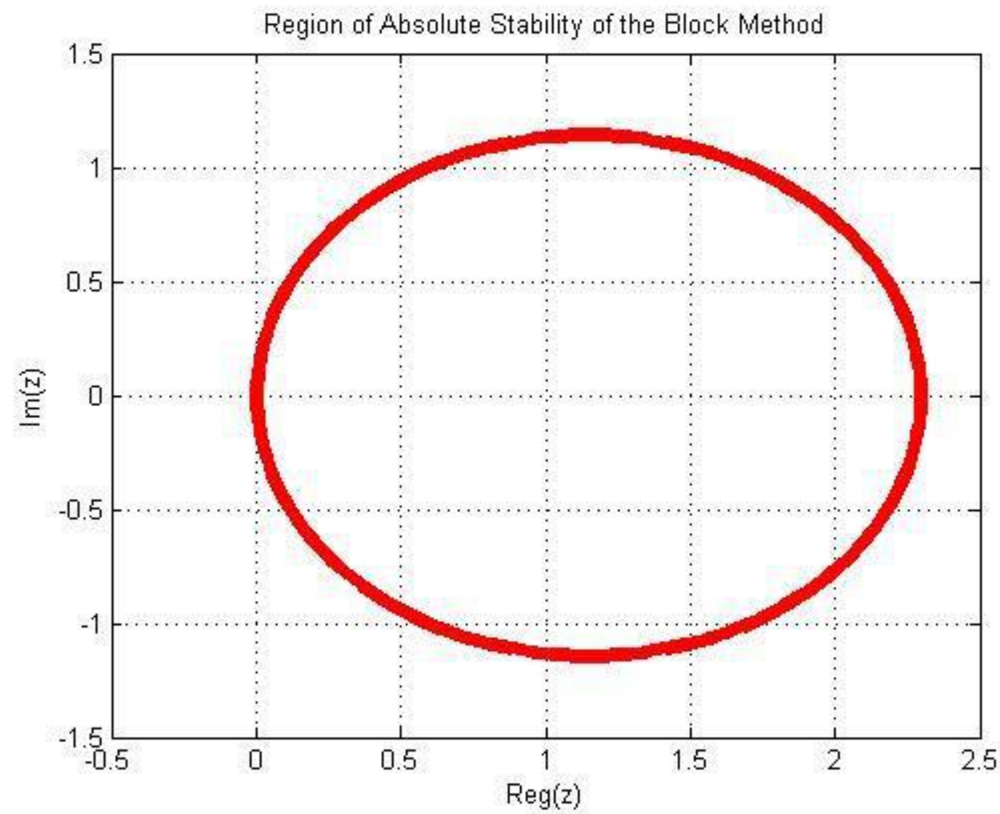

Fig.1: It shows the region of absolute stability of the developed method

V. NUMERICAL EXPERIMENT

This section deals with the implementation of the block method in solving initial value problems (IVP) of first order ordinary differential equations. The method is coded in MATLAB (R2012a) version environment using window 10 as an operating system on Acer Laptop. The new developed method is tested on some problems to determine the performance of the new proposed methods. The error is defined as Error $= |y(x) - y_n(x)|$, where $y(x)$ is the exact solution, $y_n(x)$ is the computed result.

5.1 Numerical Examples

The following problems are chosen as test problems

Problem 1: Consider the first order problem

$$y' = -y, y(0) = 1, \ 0 \le x \le 1, \ h = 0.1, \tag{20}$$

Exact solution: $y(x) = e^{-x}$

Problem 2: Consider the Prothero-Robinson Oscillatory ODE solved by Sunday et al. (2015)

$$y' = L(y - \sin x) + \cos x, y(0) = 0, L = -1, h = 0.1 \tag{21}$$

Exact solution: $y(x) = \sin x$

Problem 3: Consider the Growth Model

A bacteria culture is known to grow at a rate proportional to the amount present. After one

hour, 1000 strands of the bacteria are observed in the culture; and after four hours, 3000 strands.

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Find the number of strands of the bacteria present in the culture at time $t: 0 \leq t \leq 1$

Let N(t) denote the number of bacteria strands in the culture at time t, the initial value

problem modeling this problem is given by,

$$\frac{dN}{dt} = 0.366N, \qquad N(0) = 694 \tag{22}$$

The exact solution is given by

$$N(t) = 694e^{0366t}$$

Problem 4: Consider the SIR Model

The SIR model is an epidemiological model that computes the theoretical number of people infected with a contagious illness in a closed population over time.

The name of this class of models derives from the fact that they involve coupled equations relating the number of susceptible people S(t), number of people infected I(t) and the number of people who have recovered R(t). This is a good and simple model for many infectious diseases

Including measles, mumps and rubella. It is given by the following three coupled equations:

$$\left\{ \begin{aligned} \frac{dS}{dt} &= \mu(1-S) - \beta IS \\ \frac{dI}{dt} &= -\mu I - \gamma I + \beta IS \\ \frac{dR}{dt} &= -\mu R + \gamma I \end{aligned} \right\} \tag{23}$$

where μ, γ, β are positive parameters to be determined. Define y to be,

$$y = S + I + R$$

and adding the equations in (23) above, we obtain the following evolution equation for y,

$$y' = \mu(1-y)$$

Taking $\mu = 0.5$ and attaching an initial condition $y(0) = 0.5$ (for a particular closed population). we obtain,

$$\frac{dy}{dt} = 0.5(1-y), \qquad y(0) = 0.5$$

whose Exact solution is

$$y(t) = 1 - 0.5e^{-05t}$$

Table 1: The exact solution, computed solution, the absolute error and the time taken for the computation in (seconds) of the new method for problem 1

X-value	Exact-solution	Computed-solution	Error	Time/sec
0.10	0.9048374180359595	0.9048374164190154	1.6169441e-09	0.205696
0.20	0.8187307530779814	0.8187307469199735	6.1580079e-09	0.367746
0.30	0.7408182206817172	0.7408182076888079	1.2992909e-08	0.551484
0.40	0.6703200460356384	0.6703200243242226	2.1711416e-08	0.702339
0.50	0.6065306597126323	0.6065306278521965	3.1860436e-08	0.872062
0.60	0.5488116360940253	0.5488115930362602	4.3057765e-08	1.089429
0.70	0.4965853037914083	0.4965852486351736	5.5156235e-08	1.311644
0.80	0.4493289641172203	0.4493288965201419	6.7597078e-08	1.510724
0.90	0.4065696597405978	0.4065695792037555	8.0536842e-08	1.704233
1.00	0.3678794411714410	0.3678793476848027	9.3486638e-08	1.928272

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Table 2: The exact solution, computed solution, the absolute error and the time taken for the computation in (seconds) of the new method for problem 2

X-value	Exact-solution	Computed-solution	Error	Time/sec
0.10	0.0998334166468282	0.0998334154028488	1.2439794e-09	0.181433
0.20	0.1986693307950614	0.1986693259603136	4.8347478e-09	0.380374
0.30	0.2955202066613387	0.2955201961494993	1.0511839e-08	0.564763
0.40	0.3894183423086477	0.3894183242933311	1.8015317e-08	0.751144
0.50	0.4794255386041985	0.4794255115178666	2.7086332e-08	0.952488
0.60	0.5646424733950294	0.5646424359271566	3.7467873e-08	1.129044
0.70	0.6442176872376839	0.6442176383320178	4.8905666e-08	1.332819
0.80	0.7173560908995147	0.7173560297503054	6.1149209e-08	1.620844
0.90	0.7833269096274749	0.7833268356745610	7.3952914e-08	1.904968
1.00	0.8414709848078880	0.8414708977305652	8.7077323e-08	2.125000

Table 3: The exact solution, computed solution, the absolute error and the time taken for the computation in (seconds) of the new method for problem 3

X-value	Exact-solution	Computed-solution	Error	Time/sec
0.010	696.5446939492859400	696.5446939492180700	6.7871042e-011	0.022520
0.020	699.0987185430900600	699.0987185427908300	2.9922376e-010	0.050186
0.030	701.6621079941421600	701.6621079934537900	6.8837380e-010	0.070106
0.040	704.2348966406204900	704.2348966393841500	1.2363444e-009	0.090180
0.050	706.8171189466110000	706.8171189446453600	1.9656454e-009	0.110484
0.060	709.4088095025695100	709.4088094997416600	2.8278464e-009	0.133450
0.070	712.0100030257847300	712.0100030218745800	3.9101451e-009	0.152786
0.080	714.6207343608434700	714.6207343557549600	5.0885092e-009	0.175301
0.090	717.2410384800974700	717.2410384736124300	6.4850383e-009	0.210087
0.100	719.8709504841319800	719.8709504760998900	8.0320888e-009	0.232457

Table 4: The exact solution, computed solution, the absolute error and the time taken for the computation in (seconds) of the new method for problem 4

X-value	Exact-solution	Computed-solution	Error	Time/sec
0.010	0.5024937604036588	0.5024937604048754	1.2165824e-012	0.043527
0.020	0.5049750831254160	0.5049750831324521	7.0361494e-012	0.048093
0.030	0.5074440301984686	0.5074440302153604	1.6891821e-011	0.053913
0.040	0.5099006633466223	0.5099006633774158	3.0793479e-011	0.059570
0.050	0.5123450439858337	0.5123450440363059	5.0472182e-011	0.063933

0.060	0.5147772332257459	0.5147772332973700	7.1624151e-011	0.080116
0.070	0.5171972918712168	0.5171972919729365	1.0171974e-010	0.085281
0.080	0.5196052804238385	0.5196052805535286	1.2969015e-010	0.093241
0.090	0.5220012590834500	0.5220012592496057	1.6615576e-010	0.097912
0.100	0.5243852877496430	0.5243852879546123	2.0496926e-010	0.104638

VI. SUMMARY AND CONCLUSION

This paper examined the derivation, analysis and has demonstrated a successful implementation of seventh-eight step in form of linear multistep method. The method uses block approach for the solution of first order differential equations. The results for $y\left(x_{n+j}\right), j=0,\frac{1}{8},\frac{1}{4},\frac{3}{8},\frac{1}{2},\frac{5}{8}$ and $\frac{7}{8}$ were obtained in block forms. The new method speed up the computational processes, less burden in the implementations and also increase the rate of convergence of the solutions. The running times for the problem solved are less than four seconds as shown in Table 1-4. The exact solution of the method is in good agreement with the numerical solution. The method was found to be zero stable, consistence, convergent and P-stable as show in figure 1. Hence we conclude that seventh-eight method is reliable and gives good results with lesser time for computation of first order ordinary differential equations.

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